Special Reports

An Introduction to Listed Binary Options

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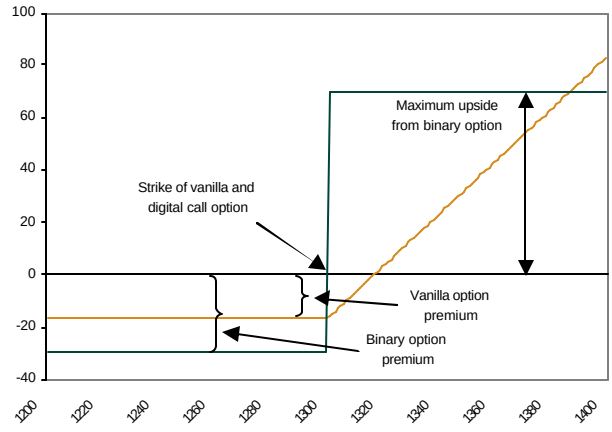
INTRODUCTION

Starting July 1, the CBOE has listed binary option contracts on the SPX and the VIX, thereby opening up a wider range of strategies for investors in the exchange-traded space. In this introduction, we describe the characteristics and payoffs of binary options, as well as specifics of the listed contracts. We suggest potential applications of binary options including expression of range views or subjective probability distributions, monetizing the implied vs realized volatility premium and constructing payoffs similar to other exotic options. We also discuss pricing and replication of these options and highlight the importance of the volatility skew in valuing them.

CHARACTERISTICS OF BINARY OPTIONS

Binary or digital options are contracts that pay out a fixed amount or nothing at expiration, depending on the settlement price of an underlying asset. The price of a binary option represents the risk neutral probability of its finishing in the money. The expiration payoff for a binary call option is shown in Figure 1 and compared with that of a vanilla call option.

Figure 1. Expiration Payoff of Binary Call Option



Source: Lehman Brothers

A key difference compared to vanilla options for the option writer is that the maximum possible downside is known in advance. This makes selling binary options a much more risk controlled and less negatively skewed strategy than the typical short volatility position.

In the past, binary options have been over the counter (OTC) instruments and have more commonly been embedded in other exotic products rather than transacted separately. The exchange traded nature of this product opens it up to a new class of investors. This is not the first instance binary options have been made available on an exchange - binary options on the target fed funds rate listed on the CBOT allow investors to express views on the probability of interest rate changes around FOMC meetings.

LISTED CONTRACT SPECIFICATIONS

Initially CBOE plans to offer only binary call options with the S&P 500[1] and VIX[2] as alternative underlyers. Option prices will range from $0 to $1, with a multiplier of 100. Thus, for a binary call trading at $0.3, each contract will involve an initial premium of $30 and a maximum potential payoff of $100 at expiration if it finishes in the money.

The minimum tick size will be $0.01 and to begin with, only the first three expirations will be available. Though only binary calls will be traded initially, these can be easily used to replicate binary put options. A long position in digital call is equivalent to a short position in a digital put, net of funding.

Intraday pricing of binary options is available on Bloomberg using the tickers BSZ Index for S&P 500 and BVZ for options corresponding to the VIX. Similar to vanilla options, the OMON function allows the user to pull up the option chain for all available maturities. We summarize some other specifics of listed binary options:

Exercise Style. The listed will be of European in nature, and can be exercised only on the last business day prior to expiration. If the settlement value of the index were to equal or exceed the strike price of the binary, exercise will be automatic.

Strike Intervals. Strikes can be at a minimum of 5-point intervals for SPX binary options and at 1-point intervals for VIX, though in the first week these have respectively commenced with 25-point and 2.5-point intervals.

Option Expiration. Both sets of options will expire on the expiration dates for the corresponding underlying indexes. Thus, SPX binary options will expire on the Saturday following the third Friday of the expiration month, with Thursday being the last trading day. For binary options on the VIX, expiration will be at the open on the Wednesday that is 30 days prior to the third Friday of the following calendar month. Determination of whether each option is in the money at expiration will be based on the settlement value for the corresponding index.

Margin Requirements. Buyers of binary options will have to pay the full premium, while option sellers will have to maintain the entire settlement amount - or $100 per contract - as margin. For purchases of binaries with more than 9 months to expiration, CBOE has a minimum margin requirement of 75% of the premium, which will become applicable once longer expirations are listed. For spreads, if the long and short have the same expiration and the exercise price of the long binary call is lower than that of the short, only the premium for the long needs to be paid and no margin is required; in other cases, both legs would be margined separately.

[1] *Please see http://www.cboe.com/products/indexopts/bsz_spec.aspx for product specifications of binary options on the S&P 500*
[2] *Details of VIX binary options are available at http://www.cboe.com/products/indexopts/bvz_spec.aspx*

TRADING STRATEGIES WITH BINARY OPTIONS

We consider several potential strategies that are implemented much more easily with binary options than traditional listed alternatives.

Trading the Implied Probability Distribution

Since the price of a binary option represents the probability of reaching that particular strike, the most obvious application is expressing views on the probability distribution of the underlying at different strike (or return) levels. For example, an investor who believes the probability of the S&P 500 closing above 1350 by the Aug08 expiration is 30% at a time when the Aug08 binary call trades at ~0.25 (corresponding to a 25% probability) should perceive some edge in buying that option.

Expressing a Range Trading View

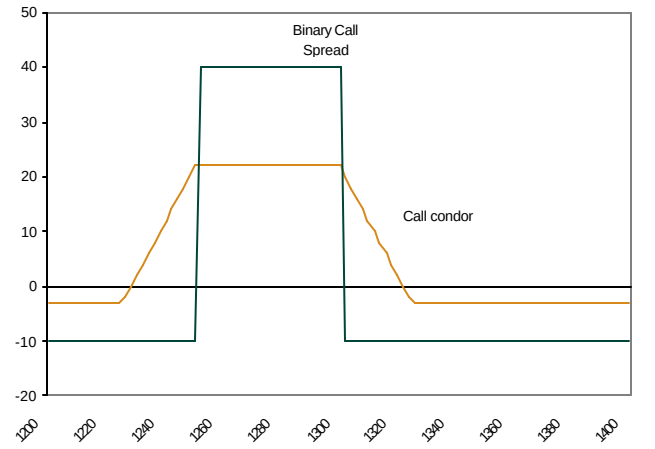
A binary option spread, such as one set up by purchasing a binary call at a given strike versus selling a binary call at a higher strike, is the cleanest way of implementing the view that the underlying remains with a defined range. Since a binary option is similar to a call spread, a binary call spread offers a risk reward similar to a condor.

For instance, with the SPX Aug08 1250 binary trading at 0.5 and the Aug08 1300 binary at 0.3, the 1250-1300 binary call spread costs $0.2 and pays out $1 if the index ends between 1250 and 1300 as of the August expiration (Figure 2). In comparison, the condor wins in a similar range but the boundaries are not as clearly defined.

Creating Contingent Premium Options

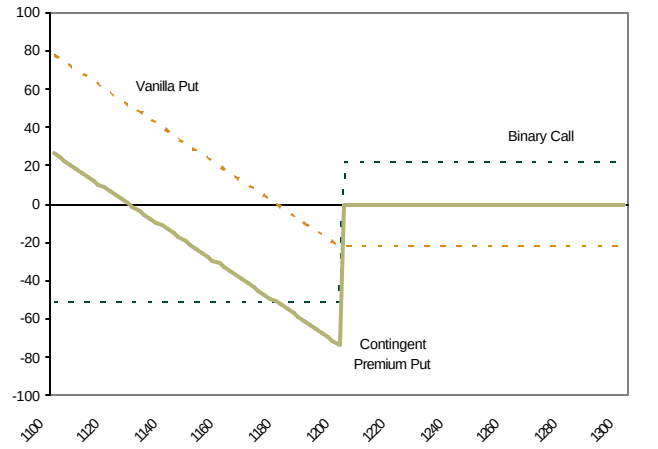
Contingent premium options are those in which the buyer pays a premium only if the option finishes in the money, and are commonly used as a cheapening mechanism. A plain vanilla option in combination with a binary option whose payoff at expiration would equal the premium of the vanilla creates a structure similar to a contingent premium option (Figure 3). Such a structure loses close to the strike in compensation for the lower premium but does not involve a premium payment if one's directional view happens to be incorrect.

Figure 2. Expressing a Range View with Binary Options



Source: Lehman Brothers

Figure 3. Contingent Premium Put Replicated with Vanilla Put and Binary Option



Source: Lehman Brothers

Creating Barrier-Like Payoffs

Barrier options, such as puts that knock out if the underlying hits a predetermined barrier, are commonly used as lower cost alternatives that also incorporate an embedded range assumption.

The discontinuous nature of the binary option's payoff makes it suitable for construction payoffs that resemble those of barrier options. Though the European nature of the binary differs from the typical continuous monitoring in the case of a barrier, the payoff at expiration can have similar characteristics.

Consider the Aug08 1250-1175 put spread on the SPX that trades for ~$28 with the index at 1252. If we add a binary call at the 1175 strike and size it so that the premium offsets the maximum upside from the put spread, the expiration payoff looks like that of a down-and-out put (Figure 4). By a similar reasoning, a payoff resembling an up-and-out call can be constructed by purchasing an SPX call spread and selling BSZ calls at the higher strike of the spread.

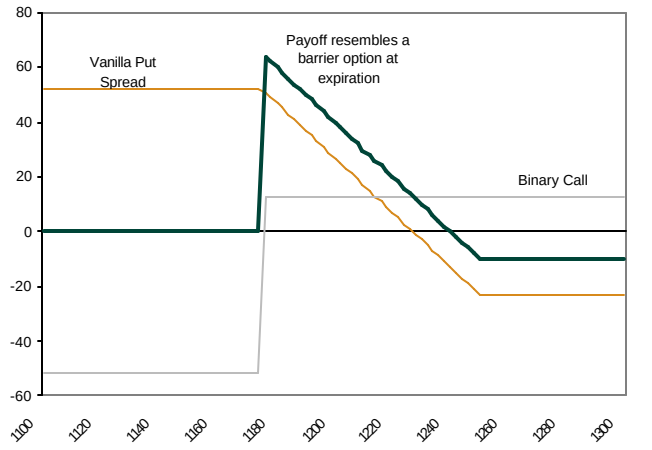
Monetizing the Implied vs Realized Volatility Premium

Traditional short volatility strategies attempt to exploit the historical premium of implied over realized volatility, especially for index options. These can take the form of systematically writing listing options or OTC instruments like variance swaps to monetize an intrinsic source of demand protection from portfolio managers.

Binary options can be an alternative avenue for collecting this premium. As we shall discuss later, the price of an in the money digital call or out of the money digital put embeds information about the level of implieds as well as skew from the volatility surface. Selling such an OTM put on a periodic basis is a substitute for conventional short gamma strategies. Figure 5 shows the cumulative performance of a strategy of selling one 10% OTM binary put (10% ITM binary call) at the SPX expiration each month. The maximum drawdowns, while larger in magnitude than the periodic premiums collected, appear much more manageable.

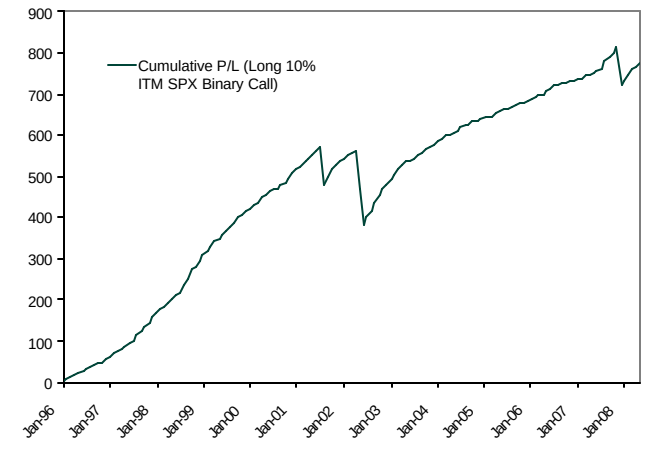
Binary options on the VIX offer a channel for taking a short position on future implied volatility as well with limited downside risk.

Figure 4. Constructing Barrier-Like Payoffs



Source: Lehman Brothers

Figure 5. Implementing a More Risk Controlled Volatility Selling Strategy



Source: Lehman Brothers, OptionMetrics

Cheapening Premium for Directional Views

The limited risk involved in selling binary options makes them suitable for writing even in unhedged form. A common tactic in vanilla space for cheapening the cost of an option struck close to the money is to write the wings against it. With binaries, the tail risk from selling deep OTM options is more controlled.

As an example, with the S&P 500 at 1252, the purchase of the Sep08 1250 put could be partially financed by writing the Sep08 1200-1325 binary strangle if the investor believes there is limited possibility of ending beyond this range.

PRICING AND HEDGING BINARY OPTIONS

For binary options with European exercise, pricing is relatively straightforward as an analytical expression is available in the Black Scholes world.

For the European binary call that pays off $1 at expiration T if the underlying S is over the strike K, the expiration payoff can be summarized as

$$c(T) = \begin{cases} 1, & S_T >= K \\ 0, & S_T < K \end{cases}$$, where S_T is the underlying at option expiration.

In a risk neutral world, the price of such an option can be determined by integrating over the range of possible price outcomes

$$c = e^{-rT} \int_K^\infty p(S)dS$$

where r is the risk free rate and $p(S)$ the probability density function. With lognormally distributed prices, $\ln(S)$ has a normal distribution with mean $\ln(S_0) + \left(r - s^2/2\right)T$ and standard deviation $s\sqrt{T}$.

Expressed in terms of the cumulative probability, the binary call price is

$$c = e^{-rT} P[S >= K]$$

or, $c = e^{-rT} P[\ln(S) >= \ln(K)]$

Using the fact that $\ln(S)$ is normally distributed, this equates to

$$c = e^{-rT}\left[1 - N\left(\frac{\ln(S_0) + \left(r - s^2/2\right)T - \ln(K)}{s\sqrt{T}}\right)\right]$$

or, $c = e^{-rT}\left[1 - N(-d_2)\right]$

or, $c = e^{-rT} N(d_2)$

Here, $N(\ )$ is the cumulative normal distribution function and d_2 is the familiar expression in the Black Scholes pricing formula, given by

$$d_2 = \frac{\ln\left(S_0 / K\right) + \left(r - s^2/2\right)T}{s\sqrt{T}}$$

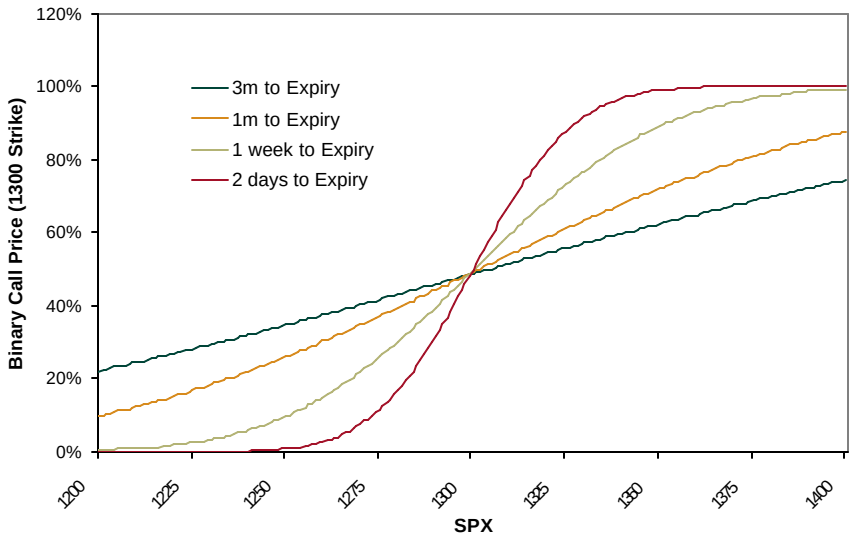
Replication with Option Spreads

The simplest listed instrument available that reasonably mimics the payoff of a binary call option is a call spread. The ideal hedge would be a spread of infinitesimal width, but even with the strike intervals available in listed options, replication of a binary is reasonably accurate except very close to expiration. The imperfectness of such a hedge is a consequence of the non-zero probability of the underlying finishing between the two strikes. Since the 5-point interval between strikes for near term SPX options is much tighter than the 1-point interval between VIX option strikes, this theoretical hedge implies a narrower range for the bid-offer spread on SPX binaries.

To illustrate the construction of a replicating call spread, we consider the 1300 strike binary call option on the SPX expiring in Aug08. A potential hedge for this is the 1295-1300 call spread on SPX with the same expiration. Since the binary pays off $1 (corresponding to a payoff of $100 per contract) if SPX closes at or above 1300 at expiration, compared to a $5 payoff for the spread, we need 0.2 units of the call spread for each binary option.

Figure 6 illustrates how the price of a binary option varies as a function of the underlying as expiration approaches. With several months to go in the life of the option, the mark to market of the option is not unlike that of the underlying itself. With about a week to expiration, it resembles a call spread with a similar time to expiration. However, very close to expiration, the binary becomes very convex just below the strike and is highly negatively convex at levels slightly above the strike.

Figure 6. Binary Option Price vs Underlying



Source: Lehman Brothers

Option Delta vs Binary Option Price

A common rule of thumb among market participants is to use a vanilla option's delta as an approximation of the probability of the option finishing in the money. While this is a reasonable approximation for names with low volatility or with expiration coming up shortly, the delta at a given strike diverges from the price of the binary option at the same price as the implied volatility increases.

To see this formally, we note that the Black-Scholes delta is given by $e^{-rT}N(d_1)$, where d_1 is given by

$$d_1 = \frac{\ln\left(S_0 / K\right) + \left(r + s^2/2\right)T}{s\sqrt{T}}$$

This equals the binary price $e^{-rT}N(d_2)$ only when $d_1 = d_2$, i.e. when $s\sqrt{T}$ equals zero.

A direct consequence of the similarity between binary option prices and vanilla option deltas is that the delta of a binary option behaves like the gamma of a regular option.

Thus, the market maker hedging the delta of the option with an opposite position in the underlying asset is likely to find the delta exploding in the vicinity of the strike as expiration approaches.

Effect of Skew

It is important to note that the convexity of a binary option changes sign depending on whether it is "in the money" or "out of the money". A binary call option is long gamma when spot trades below the strike and becomes progressively shorter gamma as the underlying moves above the strike.

Such a property has interesting consequences in assets where realized volatility depends on returns of the underlying. Thus, the behaviour of the S&P 500 that tends to become more volatile after a significant selloff and usually sees volatility dampen on rallies, would favour the buyer of a binary call. This edge needs to be explicitly adjusted for and the volatility skew taken into account when pricing binaries.

To see how the introduction of a volatility skew affects the pricing of the binary, we use fact that the binary option can be replicated by a very tight call spread. The price of a call spread as a percentage of the strike width is effectively the first derivative of the call price with respect to strike.

Using this, we can write the price of the digital call c in terms of the Black-Scholes price of a vanilla call C as

$$c = \frac{\partial C(K, s)}{\partial K}$$

By the chain rule, this is equivalent to

$$c = \frac{\partial C(K, s)}{\partial K}\frac{\partial K}{\partial K} + \frac{\partial C(K, s)}{\partial s}\frac{\partial s}{\partial K}$$

The first term here is the price of the binary assuming no volatility skew while the second term includes an adjustment for how implied volatility changes by strike $\left(\frac{\partial s}{\partial K}\right)$. Thus,

$$c = Binary_{No-skew} + Vega_{Black-Scholes} * Skew$$

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• Uncovered call writing: The risk of selling an uncovered call is unlimited and may result in losses significantly greater than the premium received.
• Uncovered put writing: The risk of selling an uncovered put is significant and may result in losses significantly greater than the premium received.
• Call or put vertical spread purchasing (same expiration month for both options): The basic risk of effecting a long spread transaction is limited to the premium paid when the position is established.
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